(6 pages)

Reg. No. : .....

Code No. : 5091 **Sub. Code : HMAE 41**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2020.

Fourth Semester

Mathematics

Elective – GRAPH THEORY

(For those who joined in July 2012-2015)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If G is simple then

(a) $\varepsilon = \binom{\gamma}{2}$ (b) $\varepsilon \geq \binom{\gamma}{2}$

(c) $\varepsilon \leq \binom{\gamma}{2}$ (d) $\varepsilon \leq \binom{\gamma - 1}{2}$

2. If e is a link of G with 5 vertices, 6 edges and 2 components then $\gamma(G.e)+\sum(G.e)+\omega(G.e)$ is

(a) 11 (b) 12

(c) 10 (d) 13

3. The edge connectivity $k^1(G)=1$ if

(a) G is connected

(b) G is connected with a cut edge

(c) G is connected with a cut vertex

(d) G is disconnected

4. The number of bridges in the Konigsberg bridge problem is

(a) 8 (b) 6

(c) 9 (d) 7

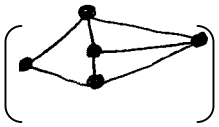
5. If every vertex of G is $M-$saturated, then the matching M is called

(a) A maximum matching

(b) A perfect matching

(c) A minimum matching

(d) A $M-$ saturated matching

6. χ^1  =?

(a) 1 (b) 2

(c) 3 (d) 4

7. If $\delta > 0$, then $\alpha - \beta' = ?$

(a) $\alpha' - \beta$ (b) $\beta - \alpha'$

(c) $\alpha' + \beta$ (d) $\alpha' - \beta'$

8. The value of $r(3,3)$ is

(a) 4 (b) 5

(c) 6 (d) 1

9. If G is $k-$ chromatic, n then G contains a subdivision of k_k. This is senown as

(a) Brooks' conjecture

(b) Hajos' conjecture

(c) Dirac's conjecture

(d) Erdos conjecture

10. Every critical graph is

(a) A clique (b) A block

(c) Complete (d) An odd cycle

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define δ and Δ in a graph G and prove that $\delta \leq 2\Sigma / \gamma \leq \Delta$.

Or

(b) If G is a tree, prove that $\Sigma = \gamma - 1$.

12. (a) Let G be a 2-connected graph with $\gamma \geq 3$. Prove that any two vertices of G are connected by at least two internally disjoint paths.

Or

(b) Show that $C(G)$ is well defined.

13. (a) Prove that a matching M in G is a maximum matching if and only if G contains no M-augmenting path.

Or

(b) Let G be a connected graph that is not an odd cycle. Prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.

14. (a) Prove that a set $S \subseteq V$ is an independent set of G if and only if $V - S$ is a covering of G.

Or

(b) Prove that $r(k,l) \leq \binom{k+l-2}{k-1}$.

15. (a) If G is k-critical, show that $\delta \geq k-1$.

Or

(b) If G is simple, prove that $\prod_k(G) = \prod_k(G-e) = -\prod_k(G.e)$ for any edge e of G.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) "A graph is bipartite if and only if it contains no odd cycle" – is it true? Justify your answer.

Or

(b) Define a cut edge with an example. Prove that an edge e is a cut edge of G if and only if it is contained in no cycle of G.

17. (a) Define the parameters k, k^1 and δ and prove that $k \leq k^1 \leq \delta$.

Or

(b) Prove that a nonempty connected graph is eulerian if and only if it has no vertices of odd degree.

18. (a) State and prove Hall's theorem.

Or

(b) State and prove Vizing's pheorn.

19. (a) Prove that $\alpha + \beta = \alpha' + \beta' = \gamma$.

Or

(b) Prove that $r(k,k) \geq 2^{k/2}$.

20. (a) State and prove Brooks' theorem.

Or

(b) If G is 4-chromatic, prove that G contains a subdivision of k_4.

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